Combinatorial Desgin Theory

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Overview

Classification of Combinatorics

Combinatorics is generally divided into four sections:

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Question: What do you think about Combinatorial Design?

Applications

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- Mathematics:

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  - Cryptography

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Exercise 2: Can you generalize the previous question? (Hint: Search about Kirkman triple system)

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Let t, k, v and λ are integer numbers such that $0 \leqslant t < k < v$ and $\lambda \geqslant 1$. A t-(v, k, λ) *block design* (abbreviated t-design) is an incidence structure of points and blocks such that the following hold: There are v points, each block contains k points, and for any t points there are exactly λ blocks that contain all these points.

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Exercise 4: Let $v > k$. Suppose that A is a set with v elements and X is all k-subsets of A. Prove that for any $0 \leqslant t \leqslant k$, we can say (A, X) is a t-$(v, k, \binom{v-t}{k-t})$ design.

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Exercise 7: Can you generalize this to a Latin cube?

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Exercise 8: Please generalize the previous example.

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For example, the *ASCII* code and its descendants give 128, 8-bit codewords representing all the upper and lower case letters and a number of other symbols; in this case $n = 8$.

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Exercise 9: Let A is the set of all non-zero binary codes with length of n. For each three distinct elements of A such as a, b, c such that $a + b + c = 0$, consider a block consisting of these three elements. Specify the parameters of this design.

Finite Geometry

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We define a geometry to consist of a set P of objects called points and a set L of nonempty subsets of P called lines that satisfy the two axioms (A1) and (A2):

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($A2$) there is a set of four points, no three of which belong to one common line.

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Designs and Projective plane

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In a ***finite projective plane****, as defined by the axioms* $(P1)$, $(P2)$, $(P3)$*, every line contains* $n+1$ *points for some parameter* n.

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Remark

A finite projective plane with parameter n is denoted $PG(2, n)$.

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Example (8)

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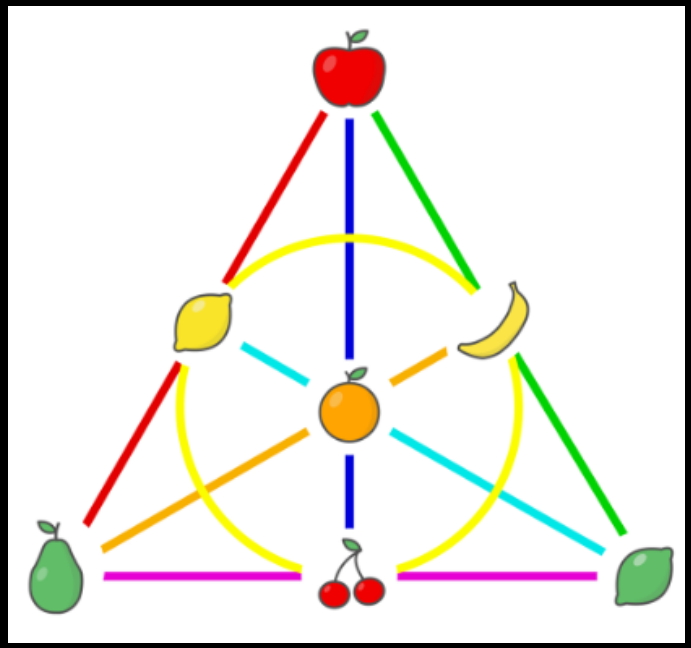

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Exercise 13: How can we make a Hadamard matrix with order n?

Theorem 1

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Let b is the number of blocks. In any t-(v, k, λ) design, we have $b = \lambda \frac{\binom{v}{t}}{\binom{k}{t}}$.

Theorem 2

Theorem (2)

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Let b, k, v and r are natural numbers such that $bk = vr$, $k < v$ and $b \leqslant \binom{v}{k}$. Then there exist a 1-(v, k, r) design with b different blocks.

Theorems 3

Theorem (3)

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Theorem (3)

In any 2-(v, k, λ) symmetric design we have:

a) $r = k$*;*

b) $\lambda(v - 1) = k(k - 1)$*; and*

c) *If B and B' are two arbitrary blocks, then $B \cap B' = \lambda$.*

Спасибо вам за внимание!