Combinatorial Designs

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Combinatorial design theory is the part of combinatorial mathematics that deals with the existence, construction and properties of systems of finite sets whose arrangements satisfy generalized concepts of balance and/or symmetry.

First of all, we will talk about a new definition: Let t, k, v and λ are integer numbers such that $0 \leqslant t < k < v$ and $\lambda \geqslant 1$. A t-(v, k, λ) **block design** (abbreviated t-design) is an incidence structure of points and blocks such that the following hold: There are v points, each block contains k points, and for any t points there are exactly λ blocks that contain all these points.

After looking at a few simple examples, let's look at a few more serious examples, such as: Graphs, Latin squares, MOLS, Binary codes, Hadamard matrices and so on.

At the end we will state three nice theorems in combinatorial designs. Then we prove them together.

Theorem 1. Let b is the number of blocks. In any t-(v, k, λ) design, we have $b = \lambda \frac{\binom{v}{t}}{\binom{k}{t}}$.

Theorem 2. Let b, k, v and r are natural numbers such that $bk = vr$, $k < v$ and $b \leqslant \binom{v}{k}$. Then there exist a 1-(v, k, r) design with b different blocks.

If in 2-(v, k, λ) design $b = v$, then it is called a **symmetric block design**.

Theorem 3. In any 2-(v, k, λ) symmetric design we have:

a) $r = k$;

b) $\lambda(v - 1) = k(k - 1)$; and

c) If B and B' are two arbitrary blocks, then $B \cap B' = \lambda$.